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THE NUMERICAL SOLUTION OF STOCHASTIC DIFFERENTIAL EQUATIONS

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Abstract

A method is proposed for the numerical solution of Itô stochastic differential equations by means of a second-order Runge–Kutta iterative scheme rather than the less efficient Euler iterative scheme. It requires the Runge–Kutta iterative scheme to be applied to a different stochastic differential equation obtained by subtraction of a correction term from the given one.

It was observed by Wright [8] that different iterative schemes for the numerical solution of stochastic differential equations

$$dx_t = a(t, x_t)\,dt + b(t, x_t)\,d\xi_t, \tag{1}$$

where ξ_t is a Wiener process, converge to different solutions for the same noise sample and initial condition. This is in contrast to their deterministic counterparts for ordinary differential equations, which converge to the same solution.

Strictly speaking stochastic differential equations (1) are really integral equations

$$x_t = x_0 + \int_{t_0}^{t} a(s, x_s)\,ds + \int_{t_0}^{t} b(s, x_s)\,d\xi_s, \tag{2}$$

where the second integral is either an Itô or Stratonovich stochastic integral, in the definitions of which the functions are evaluated, respectively, at the left-hand endpoint and midpoint of each partition subinterval (for example, [4], [6], [7, chapter 3]). Consequently the Euler iterative scheme

$$x_{n+1} = x_n + a(t_n, x_n)\,h + b(t_n, x_n)\,\xi_n \tag{3}$$

converges to a sample path of the Itô solution of (2) whereas the second-order Runge–Kutta iterative scheme

$$\begin{aligned} x_{n+1} = x_n &+ \tfrac{1}{2}\{a(t_n, x_n) + a(t_{n+1}, x_n + a(t_n, x_n)\,h + b(t_n, x_n)\,\xi_n)\}\,h \\ &+ \tfrac{1}{2}\{b(t_n, x_n) + b(t_{n+1}, x_n + a(t_n, x_n)\,h + b(t_n, x_n)\,\xi_n)\}\,\xi_n \end{aligned} \tag{4}$$

converges to a sample path of the Stratonovich solution of (2). In these iterative schemes a partition $t_0 < t_1 < \ldots < t_N$ is used with uniform step length $h = t_{i+1} - t_i$ and an $N(0, h^{\frac{1}{2}})$ distributed noise sample $\xi_0, \xi_1, \ldots, \xi_{N-1}$, where ξ_i represents the

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If you were to obsession such a