

INTRODUCTION TO TOPOLOGY (MA30055)  
SEMESTER 2 MATHEMATICS: PROBLEM SHEET 1: SOLUTIONS

1. True or false?

- (a) In the discrete topology  $\mathcal{F} = Y$ , for all  $Y \subseteq X$ .  
 (b) In the trivial topology  $\mathcal{F} = X$ , for all  $Y \subseteq X$ . [Hint: is your answer *always* true?]  
 (c) If a topological space  $X = A \cup B$ , with  $A, B$  disjoint open subspaces, then one of  $A$  or  $B$  is empty.  
 (d) Let  $f: X \rightarrow Y$  be a map between topological spaces. Suppose that, whenever  $F$  is closed in  $X$ , the image  $f_*(F)$  is closed in  $Y$ . Then, whenever  $U$  is open in  $X$ ,  $f_*(U)$  is open in  $Y$ .

**Solution:**

- (a) True: every subspace is closed.  
 (b) False:  $\emptyset = \emptyset$ , but true for non-empty subspaces.  
 (c) False: e.g.  $X = \{0, 1\}$  with the discrete topology,  $A = \{0\}$ ,  $B = \{1\}$ .  
 (d) False: remember that  $f_*(U^c) \neq (f_*(U))^c$  (it's  $f_*(U^c) = (f_*(U))^c$ ). This statement does hold whenever  $f$  is bijective. Options for a counterexample include:  
 (•) any map between cofinite topological spaces preserves closed, i.e. finite, subspaces under direct image, but not necessarily open ones.  
 (•) Let  $\Sigma$  be the Sierpiński space and define  $f: \Sigma \rightarrow \Sigma$  by  $f(x) = 0$  for all  $x \in \Sigma$ . Note that the converse is not true either: a map that preserves open subspaces under direct image does not necessarily preserve closed subspaces.

2. Show that, if  $X$  is a topological space and  $Y \subseteq X$ , then  $X \setminus Y^\circ = \overline{X \setminus Y}$  and  $X \setminus \overline{Y} = (X \setminus Y)^\circ$ .

**Solution:**  $X \setminus Y^\circ$  is closed and contains  $X \setminus Y$  (because  $Y^\circ$  is open and  $Y^\circ \subseteq Y$ ). Therefore  $X \setminus Y^\circ \supseteq \overline{X \setminus Y}$ . On the other hand,  $X \setminus (X \setminus Y^\circ) = Y^\circ$ , and  $X \setminus \overline{X \setminus Y}$  is open and contained in  $Y$ , so it is contained in  $Y^\circ$ . Therefore  $X \setminus \overline{X \setminus Y} \subseteq X \setminus (X \setminus Y^\circ)$ , so, taking complements,  $\overline{X \setminus Y} \supseteq X \setminus Y^\circ$ .

3. Recall that  $N$  is a neighbourhood of  $x \in X$  if there is  $U \in \mathcal{T}_X$  with  $x \in U \subseteq N$ . Prove the following properties.

- (a) If  $N$  is a neighbourhood of  $x$  and  $N \subseteq N'$ , then  $N'$  is a neighbourhood of  $x$ .  
 (b) If  $N_1$  and  $N_2$  are neighbourhoods of  $x$ , then so is  $N_1 \cap N_2$ .  
 (c)  $x \in \overline{Y}$  if and only if every neighbourhood of  $x$  intersects  $Y$ .

**Solution:** (a) If  $x \in U \subseteq N$  with  $U$  open, then  $x \in U \subseteq N'$ . Thus  $N'$  is a neighbourhood of  $x$ .

Click here to access this Book :

**FREE DOWNLOAD**







# Some Topology Problems And Solutions General Topology

## [Some Topology Problems And Solutions](#)

Eventually you will definitely experience additional and capacity by spending more money. again when? achieve you consent that you need to get these all needs then have a lot of money? Why dont you try get something fundamental in the beginning? This is something that will guide you to understand even more pretty much the globe, the experience, some places, soon history, fun and more?

This is your completely own epoch to feign revision of habits. along with some guides

that you might enjoy now are [Some Topology Problems And Solutions General Topology](#) below.

, [For Beginners A Practical To Reading The Cards, And Emotions Three Volume Set Complete Edgar Cayce Readings Vol 13 14 15, Read 4 Answer Key, Frank Penguin Readers, Eli Readers Uncle Jack, Wallpaper Active Reading Chart Answers, Managing Diversity Readings Cases Exercises, Story Andes Survivors Read Piers, Attendance Spreadsheet, Wants To Be A Star Penguin Readers Easy Starts, Biology Reading Fred And Theresa Holtzclaw Answers Chapter 8, Grade Reading Comprehension Workbooks](#) .

[Reading Note Taking Teachers Edition The World And Its People Western Hemisphere Europe And Russia, Reading Assessment Ks1 Mark Scheme Fishing For Fun, Reader Uninstall, Key Pathways 2 Reading And Writing, Past Puritan Devotional Readings Richard, Biology Reading Fred And Theresa Holtzclaw, Threat Orca Young Readers, Skills For Reading 3, Encounters Level 4 Students Book Reading And Writing Human Behavior Book Mediafile Free File Sharing, Adult Eli Readers Leben, Acrobat Reader For Files, Reading Second Edition, Reading Night Chapters 6 Through 9 Hopelessness And Death Answers, Read Bengali Choti Bengali Choti Bengali Choti, To Norton Reader Questions, Key Reading Explorer 3 Unit 2, Politics Classic Contemporary Readings Cigler, Reads Classic Edition The United States In Literature Testbook](#)